INVESTIGATING THE BEHAVIOUR OF TWO-DIMENSIONAL FINITE ELEMENT MODELS OF COMPOUND CHANNEL FLOW

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ABSTRACT

This paper describes results from a recent study concerning the numerical modelling of compound channel flow using two generalized two-dimensional finite element codes specifically adapted to floodplain studies: RMA-2 and TELEMAC-2D. By application to an 11 km reach of the River Culum, Devon, UK, simulations are developed to investigate the impact of numerical technique, mesh resolution and topographic parameterization on model results. The research is shown to raise a number of issues concerning the construction, calibration and validation of two-dimensional finite element models for this flow problem. © 1997 by John Wiley & Sons, Ltd.

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INTRODUCTION

Models of sediment transport, water quality and geomorphology in fluvial floodplain environments (see for example James, 1985; Pizzuto, 1987; Marriot, 1992) rely on a realistic description of flow field characteristics in order to achieve acceptable results. Recently, a number of studies have shown the potential for high resolution two-dimensional modelling of floodplain flows to provide this information (see for example Bates *et al*., 1992; Feldhaus *et al*., 1992; Anderson and Bates, 1994). Whilst model validation data have been presented in these studies, it can often be difficult to interpret in the absence of a detailed understanding of model response to parameterization uncertainty or calibration procedures. Such studies have therefore demonstrated the need for a more detailed analysis of possible errors associated with two-dimensional flow modelling techniques if such simulations are to be used to drive sediment transport or water quality calculations. We now review the development of modelling schemes to simulate overbank flow in order to identify those areas where more detailed investigation of model behaviour is required.

Extensive, often hydraulically rough, areas of low bed slope adjacent to river channels which are periodically inundated at high flows are typical of many lowland river systems. During flood events these floodplains may act either as temporary stores for water or provide an additional route for flow conveyance. The fundamental analytic problem for hydraulic models of such environments is therefore simulation of the covering and uncovering of floodplain areas caused by the downstream propagation of a low amplitude flood wave. Modelling approaches to this problem have typically employed (Samuels, 1990) finite difference solutions of the one-dimensional St. Venant equations (see for example Cunge *et al*., 1980; Samuels, 1983; Fread, 1985). Such models consider the conservation of momentum between two cross-sections Δx apart to yield a first-order partial differential equation of the form:

$$\frac{\delta Q}{\delta t}+\frac{\delta(Q^2/A)}{\delta x}+gA\left[\frac{dh}{\delta x}+S_f\right]=0 \qquad (1)$$

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where Q = flow discharge; A = flow cross-sectional area; g = acceleration due to gravity; h = depth of flow; S_f = friction slope. Similarly, the mass continuity is given as:

$$\frac{\delta Q}{\delta t} + \frac{\delta A}{\delta t} = 0 \qquad (2)$$

This constitutes a pair of simultaneous equations in Q and h which are typically solved via a finite difference approximation procedure such as the Preissmann (1961) or Abbott and Ionesco (1967) schemes.

One-dimensional finite difference models are now reasonably well understood and can be calibrated to obtain satisfactory stage predictions at specified cross-sections. This water stage prediction may then be interpolated between cross-sections, against known topographic data, to give the inundation extent. Recently, computing and numerical advances have, for the first time, made two-dimensional finite element solutions to this flow problem a realistic proposition. Such an approach treats the computational domain as a continuous field rather than as a series of cross-sections and draws on the flexibility of the finite element space decretization to represent complex topographies, such as a channel meandering within a wider floodplain belt, with a minimum number of elements. Two-dimensional methods can predict a wide variety of flow variables, including velocity vectors and depths at each computational node as well as simulating flood inundation extent. As the finite element grid effectively constitutes a continuous independent Digital Terrain Model, all flow effects over topography are inherently included and no further interpolation is necessary to derive the inundation extent. Two-dimensional finite element models would therefore appear to have significant potential with regard to the simulation of river channel/floodplain flow which merits full investigation.

Two-dimensional finite element models as originally developed for application to free surface flow problems are of limited use for floodplain studies due to their inability to realistically incorporate dry areas within the computational domain. In such models representation of the flow field boundary can only be achieved by including or eliminating partially wet elements from the solution domain as a whole (see Figure 1).

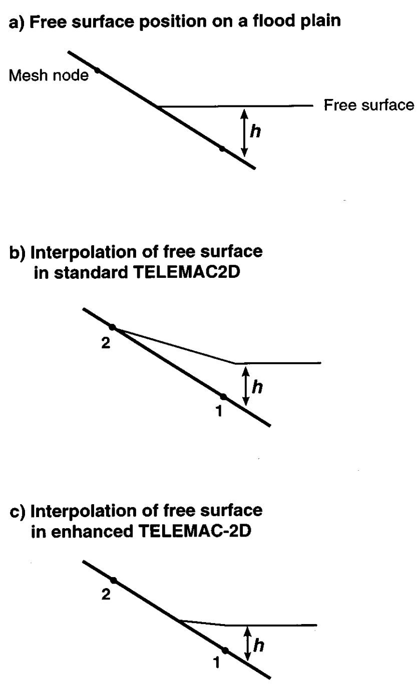


Figure 1. (a) Free surface position on a partially wet element. (b) Typical finite element interpolation of a free surface. (c) Interpolation achieved using the enhanced wetting and drying algorithm in TELEMAC-2D

Thus, the floodplain inundation extent is represented as an irregular front based on the element geometry rather than as a smooth feature. Undesirable, and potentially mathematically unstable, oscillations in the frontal position or spurious flow velocities may also occur with such schemes in response to relatively small depth changes. Research in this area (Lynch and Gray, 1980; King and Roig, 1988) has led to the development of two generalized two-dimensional finite element codes that have the potential to simulate the covering and uncovering of areas of low lateral bed slope in a realistic fashion: the RMA-2 code (King and Norton, 1978), originally developed for the US Army Corps of Engineers, and the TELEMAC-2D code, developed by the Laboratoire National d'Hydraulique, Chatou, France. The generalized RMA-2 code has been further developed (Gee *et al*., 1990; Bates *et al*., 1992) specifically to simulate river channel/floodplain flows and initial studies made applying the RMA-2 code to the River Fulda in Germany (Baird *et al*., 1992) and the River Culm, Devon, UK (Bates and Anderson, 1993; Anderson and Bates, 1994). These have demonstrated a number of topographic constraints on the application of the RMA-2 code to floodplain studies. More recently, TELEMAC-2D has also been enhanced (Hervouet and Janin, 1994; Bates *et al*., 1994) to provide a topographically robust high space/time resolution flood inundation modelling capability. RMA-2 and TELEMAC-2D therefore currently represent the only two-dimensional finite element models that have been specifically adapted for reach scale floodplain flow simulations. Two-dimensional finite element modelling of floodplain flows has been shown to be viable, giving model predictions broadly in line with field data and a logical response to simple sensitivity analysis tests. The initial studies have, however, demonstrated the need for a more detailed analysis of model numerical solution and parameterization in a number of specific areas which we address in this paper:

1. investigation of the impact of alternative numerical solution techniques;
2. examination of the impact of mesh resolution on model simulation results;
3. initial quantification of the impact of topographic parameterization accuracy.

These issues currently represent those most poorly understood in relation to this flow modelling problem yet can potentially have a significant impact on model results. We argue here that a fuller understanding of the behaviour of the RMA-2 and TELEMAC-2D models for reach scale floodplain flows is an essential prerequisite to the interpretation of theoretical or field based model validation studies. If the interaction between the model numerical solution chosen and the derived parameter base is not made explicit, comparison of simulation results with validation data may be less convincing as scientific evidence than would otherwise be the case. In this situation the extension of such simulations to drive distributed geomorphology and water quality models may not be possible despite the obvious research benefits that would stem from such a development. We now provide a description of the RMA-2 and TELEMAC-2D models prior to detailing research conducted to explore the three issues raised above.

DESCRIPTION OF THE SIMULATION MODELS

RMA-2 and TELEMAC-2D both solve second-order partial differential equations for depth averaged fluid flow derived from the full three-dimensional Navier–Stokes equations. This gives an equation set consisting of an equation for mass continuity (3) and two force–momentum equations ((4) and (5)). These are given in non-conservative form as:

$$\frac{\delta u}{\delta t} + \vec{u}.\overrightarrow{\mathrm{grad}}(h) + h\,\mathrm{div}(\vec{u}) = 0 \tag{3}$$

$$\frac{\delta u}{\delta t} + \vec{u}.\overrightarrow{\mathrm{grad}}(u) + g\frac{\delta h}{\delta x} - \frac{1}{h}\mathrm{div}(\mathrm{v}\,h\,\overrightarrow{\mathrm{grad}}(u)) = S_x - g\frac{\delta Z_f}{\delta x} \tag{4}$$

$$\frac{\delta v}{\delta t} + \vec{u}.\overrightarrow{\mathrm{grad}}(v) + g\frac{\delta h}{\delta y} - \frac{1}{h}\mathrm{div}(\mathrm{v}\,h\,\overrightarrow{\mathrm{grad}}(v)) = S_y - g\frac{\delta Z_f}{\delta y} \tag{5}$$

where u,v = velocity components in the x and y cartesian directions; h = depth of flow; Z_f = bed elevation; ν = turbulent viscosity; S_x,S_y = source terms; g = gravitational acceleration; t = time.

The models therefore solve for the three unknowns h, u and v. Both models employ a mean flow concept to treat turbulent flows, averaging instantaneous velocities over time to give mean motion only. In this formulation an additional term, the Reynolds stress, is added to the governing equations to represent the increased internal shear stress on mean flow produced by velocity fluctuations. Evaluation of the Reynolds stress term poses a number of difficulties for field applications. To estimate this additional unknown and make the governing equations mathematically tractable, some model of the turbulence is therefore introduced. In both models employed in this study the Boussinesq approximation is adopted. Here the Reynolds stress is assumed to be the product of the depth–mean velocity gradient and an exchange coefficient, ε, dimensionally similar to the coefficient of viscosity, μ, and termed the eddy viscosity. This eddy viscosity term can then be used to parameterize the model or can be estimated through a further equation set, such as the k–ε model (Guiyi, 1992). For both models used in this study these additional terms were derived from a constant eddy viscosity model.

Specific attributes of the RMA-2 code

RMA-2 solves the shallow water equations for a continuum of triangular and quadrilateral elements using a fully implicit implementation of the Galerkin weighted residual technique. This discretization is linear for water depth and quadratic for flow velocity, giving six or eight nodes per triangular or quadrilateral element respectively. RMA-2 therefore allows meshes which contain a mixture of triangular and quadrilateral element geometries. Due to the extreme non-linearity of the governing equations the numerical integration for the Galerkin procedure is performed iteratively using a Newton–Raphson type solver (Norton *et al*., 1973). RMA-2 fully assembles the matrices of the linear system at each iteration. As these matrices are both large and sparse this can add significantly to the time required to perform a particular calculation, although implementation of a suitable element reordering scheme, such as the King (King, 1970) or Sloan (Sloan and Randolph, 1982) algorithms, can improve this position by reducing the matrix frontwidth.

In the original RMA-2 model, partially wet elements occurring during floodplain wetting and drying are eliminated from the solution domain if the water depth at any node within an element becomes less than zero. This so-called ‘step’ formulation unrealistically represents the flow boundary as an irregular wetting and drying front. To overcome this, the model has been upgraded to include a specific wetting and drying algorithm (King and Roig, 1988). This method modifies the governing equations for partially wet elements to simulate a smooth transition between wet and dry states by defining a domain coefficient, θ, representing the proportion of an element available for fluid flow. To approximate the flow boundary, partially wet elements are retained within the solution and the domain coefficient used to scale the simulated elemental water volume to the true volume residing on the element at each time step.

Specific attributes of the TELEMAC-2D code

To solve Equations (3), (4) and (5) TELEMAC-2D employs a fractional step method (Marchuk, 1975) where advection terms are solved initially, separate from propagation, diffusion and source terms, which are solved together in a second step. This is achieved for a space discretization consisting of either linear triangular or bi-linear quadrilateral elements with three of four nodes per element respectively. Several schemes may be used for the advection step, with the Method of Characteristics chosen here for the momentum equation. To ensure mass conservation, two alternative schemes are available for the advection of h in the continuity equation: the Streamline Upwind Petrov Galerkin (SUPG) method (Brookes and Hughes, 1982) and a hybrid numerical method specifically developed for TELEMAC-2D. According to the SUPG technique, standard Galerkin weighting functions are modified by adding a streamline upwind perturbation. The hybrid numerical scheme consists of a combination of the Method of Characteristics and centred differences, which sacrifices the unconditional stability of the Method of Characteristics for an improvement in mass conservation properties. The second step (propagation) of TELEMAC-2D makes use of an implicit time discretization and solves the resulting linear system with a conjugate gradient-type method. In addition, and unlike RMA-2, the TELEMAC-2D code makes significant savings in both computational time and storage requirements through

the use of an element-by-element solution technique. Here the matrices in the linear system are stored in their elementary form without recourse to full assembly.

TELEMAC-2D incorporates an additional modification to simulate areas of low lateral bed slope such as floodplains or tidal flats. If this option is not implemented, partially wet elements are retained within the solution domain and the model interpolates a spurious non-zero lateral free water slope across the element (see Figure 1b). For such an element the driving terms in the momentum equation are:

$$\frac{\delta u}{\delta t} = -g\frac{\delta z}{\delta x} \tag{6}$$

where z = free surface elevation.

Other terms, for example friction, diffusion etc., still act on such an element but have been discarded for clarity. On dry elements a fictitious free surface slope is computed which can cause the numerical scheme to overpredict flow velocities. The object of the modification for floodplain areas in TELEMAC-2D is to provide a better approximation of the water surface slope. First, a check is made for each element to test for the existence of a wet/dry boundary. If the bottom elevation for a particular node, Zf_2 with depth h_2, is greater than the water surface elevation at one of the other nodes in that element with depth h_1, then a new water surface elevation (z_2) is defined as:

$$z_2 = Zf_1 + h_2 \tag{7}$$

This can then be used to develop more realistic flow velocities in floodplain areas (see Figure 1c). Moreover, as only the momentum equation is modified, mass conservation properties are not affected.

A comparison of the numerical solution techniques employed by these models shows the TELEMAC-2D scheme to be theoretically more efficient than RMA-2 due to its use of an element-by-element solver. Such schemes have been shown to result in significant savings in both computational time and core storage (Binley and Beven, 1992; Hervouet, 1993). The TELEMAC-2D model therefore has the potential to solve flow problems requiring large numbers of finite elements and at a high time resolution. In addition, advanced

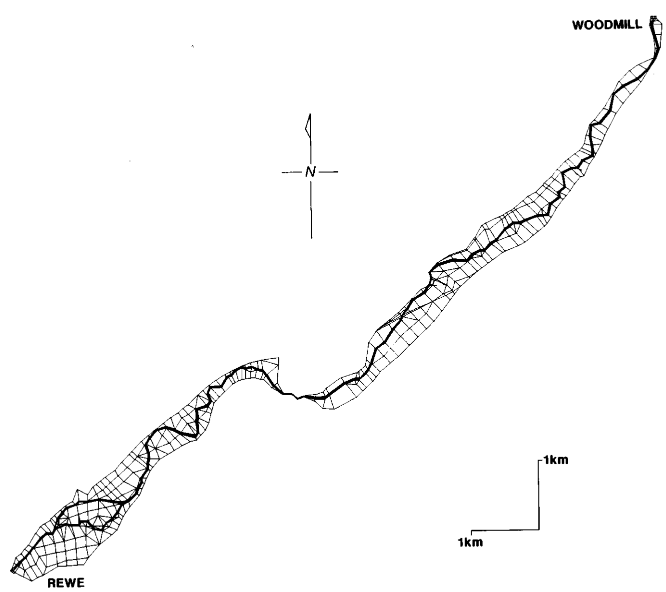


Figure 2. Low resolution finite element mesh of the River Culm, Devon, UK, for an 11 km reach between the flow gauging stations at Woodmill (upstream) and Rewe (downstream)

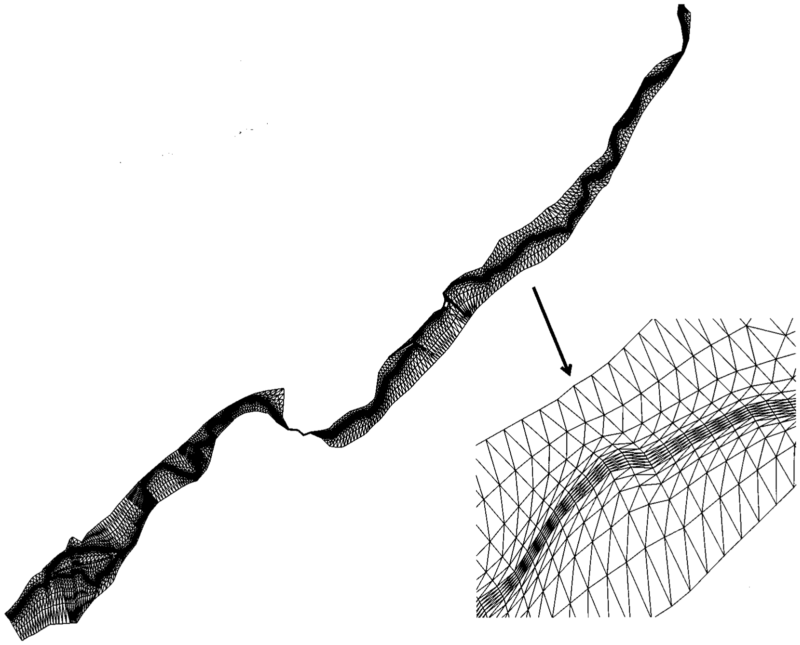


Figure 3. High resolution finite element mesh of the River Culm, Devon, UK, between Woodmill and Rewe

numerical techniques, such as the SUPG method, may confer a number of advantages in terms of accuracy and computational cost over standard Galerkin weighted residual solvers.

RESEARCH DESIGN AND NUMERICAL RESULTS

Examination of the three areas of interest outlined in the introduction to this paper was achieved in two parts. Firstly, a combined research design was devised to simultaneously analyse the impact of numerical solution technique and mesh resolution using the RMA-2 and TELEMAC-2D models. Secondly, an initial attempt was made to quantify the impact of topographic parameterization with a distributed space/time sensitivity analysis using the RMA-2 model only.

An 11 km reach of the River Culm, Devon, UK, between established stage recorder gauging stations at Woodmill and Rewe (see Figure 2) was selected as the test site for these simulations. For this reach two triangular finite element discretizations at low (see Figure 2) and high (see Figure 3) mesh resolutions were constructed. These were used for simulations with both numerical models in order to control for mesh resolution effects. The low resolution mesh discretization was constructed to minimize computational effort, which in the case of RMA-2 also involved amalgamation of a number of triangular element pairs into quadrilaterals to give a mix of triangular and quadrilateral elements. This was not expected to have a discernible impact on model results aside from an improvement in computational efficiency as the number and position of computational nodes remained constant. Due to the constraint on the number of elements, however, both RMA-2 and TELEMAC-2D low resolution meshes exhibit a high degree of distortion and no smooth transition between elements of varying size, particularly in near-channel areas. A further high resolution discretization was therefore constructed by decreasing the length-to-breadth ratio of channel and near-channel elements from approximately 20:1 to 5:1. In order to obtain a comparison between mesh properties, the Courant number, a numerical scheme stability criterion, was calculated for each mesh/model combination using the equation:

$$C_r = u\frac{\Delta t}{\Delta x} \tag{8}$$

where C_r = Courant number; x = mesh size.

Typically for an explicit scheme instability will occur if this number exceeds 1. For implicit schemes of the type presented in this paper this condition does not apply; however, the Courant number is still a useful indication of the quality of the solution with high values (>50) representing potential problems (Hervouet and Janin, 1994). A summary of the characteristics of the three finite element meshes is given in Table I. Bed elevation data sets for both discretizations were interpolated from a single topographic parameterization based on data from UK Ordnance Survey 1:2500 series maps enchanced with six surveyed cross-sections taken at the up- and downstream limits of the reach and at intermediate points where the topographic map data coverage was inadequate. This coverage enabled bottom-of-channel and top-of-bank elevations to be established at 17 locations downreach. These had a relatively even spacing and thus allowed a reasonable approximation to the channel slope to be obtained. In other parts of the domain it was relatively easy from contour information to determine the elevation of floodplain side slopes; however, topographic information in low-lying floodplain areas was sparse apart from at specific features such as the railway embankment in the central part of the domain. This gave a range of downreach surface gradients of between 0·001 and 0·0017. Lateral gradients were much smaller given the nature of the floodplain environment being of the order of 0·0001.

Table I. Characteristics of the finite element meshes generated for the River Culm, Devon, UK. The maximum Courant number for each mesh is calculated using Equation (8) on the basis of the typical time step duration for each model: 0·5 h for RMA-2, and 2 s for TELEMAC-2D

Mesh	No. of elements	No. of nodes	Mean channel node spacing (m)	Mean floodplain node spacing (m)	Mesh distortion (length to breadth ratio)	Max. Courant no.
Low resolution/ RMA-2	1090	1200	80	100	20:1	540
Low resolution/ TELEMAC-2D	2040	1200	80	100	20:1	0·6
High resolution/ TELEMAC-2D	9800	5600	25	60	5:1	0·2

Alternative numerical solution strategies and mesh resolution impacts

The first phase of the research design consisted of the simulation of a single flood event using RMA-2 and two different TELEMAC-2D models which differed in the numerical scheme chosen for the advection of h in the continuity equation. Specifically, the chosen schemes were the Streamline Upwind Petrov Galerkin and the hybrid Method of Characteristics scheme specifically developed for TELEMAC-2D. These three models were tested at each mesh resolution to give a total of six calculations in all. A one in one year recurrence interval flood event which took place on 30 January 1990 was simulated. TELEMAC-2D boundary condition data were obtained from water stage recorders at the up- and downstream ends of the reach. At the upstream boundary stage recorder (Woodmill on Figure 2) a dynamic flow rate boundary condition was derived using the established stage–discharge rating curve for this site while at the downstream boundary (Rewe on Figure 2) a water surface elevation condition was imposed. Separate calibrated boundary friction factors were applied in main channel and floodplain areas for all model simulations to account for differential roughness effects. In the case of TELEMAC-2D, Manning's n friction factors of 0·025 for the main channel and 0·083 for the floodplain were chosen as representative after a number of exploratory simulations with the low resolution mesh. This calibration was then transferred directly to the high resolution mesh. Simulations were begun from a near-

steady-state condition and consisted of 27,000 time steps of 2 s duration. This appeared to be the optimum for minimizing computational time.

RMA-2 calculations performed using the flood event described above also employed an imposed flow rate as the upstream boundary condition and began from a near-steady-state condition. Simulations did, however, differ in terms of downstream boundary condition, friction factor and time step. Here a stage–discharge relationship was used as the downstream boundary condition. This was used to define a nominal flow rate which was then treated in an identical manner to an imposed flow boundary condition. Calibrated boundary friction parameterizations were also based on the Manning's *n* coefficient, again differentiating between main channel, $n = 0{\cdot}03$, and floodplain, $n = 0{\cdot}1$. Both RMA-2 and TELEMAC-2D parameterizations are within physically realistic limits for this type of environment (Acrement and Schneider, 1984). The friction calibrations are broadly consistent, with the differences in roughness parameterization, reflecting the fact that in this study we compare best fit calibrations which give minimum phase error between predicted and observed hydrograph peaks for each model. If a single calibration were used, one model would have an implicit advantage and the comparative nature of the present study would be invalidated. Finally, as both RMA-2 and TELEMAC-2D use a fully implicit solver, no stability problems relating to Courant number were experienced. For RMA-2 a large, 1800 s (or 0·5 h), time step could therefore be employed. This had the advantage of minimizing the adverse impact on computational time resulting from full assembly of the matrices of the linear system. This does, however, result in a high value for the Courant number (see Table I) which can potentially indicate problems with the solution of the controlling equations (Hervouet and Janin, 1994).

Results for the simulations described above are given in Figures 4a and b for model predictions of downstream outlet discharge and flood inundation extent respectively. Results for simulations using the TELEMAC-2D hybrid numerical scheme on the low resolution mesh and RMA-2 on the high resolution mesh

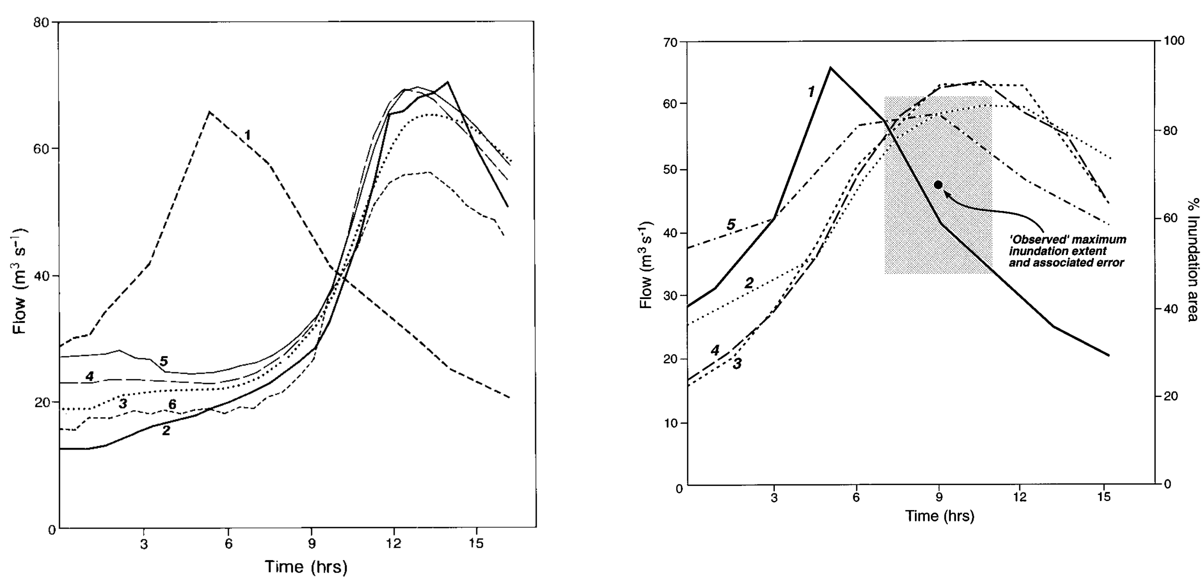


Figure 4. (a) A comparison of up- and downstream observed discharge at the Woodmill and Rewe gauging stations for a one in one year recurrence interval event with RMA-2 and TELEMAC-2D predictions, where: 1 = observed upstream discharge at Woodmill; 2 = observed downstream discharge at Rewe; 3 = predicted discharge using the SUPG method and a low resolution mesh; 4 = predicted discharge using the SUPG method and a high resolution mesh; 5 = predicted discharge using a hybrid numerical method and a high resolution mesh; 6 = predicted discharge using RMA-2 and a low resolution mesh. (b) Model predicted floodplain inundation extent for the one in one year recurrence interval flood event shown in (a), where: 1 = observed upstream discharge at Woodmill; 2 = predicted inundation extent using the SUPG method and a low resolution mesh; 3 = predicted inundation extent using the SUPG method and a high resolution mesh; 4 = predicted inundation extent using a hybrid numerical method and a high resolution mesh; 5 = predicted inundation extent using RMA-2 and a low resolution mesh

were unsuccessful and are not therefore shown. TELEMAC-2D simulations employing the hybrid numerical scheme on the low resolution mesh displayed a number of problems including poor mass conservation properties, irregular velocity vectors and significant negative depths. The method was therefore rejected as an appropriate numerical technique for use with such highly distorted meshings. In the case of RMA-2 simulations on the high resolution mesh, the computer storage requirments generated by the necessity to fully assemble matrices at each time step significantly exceeded available memory on a high powered workstation, even when an accepted frontwidth reduction algorithm (King, 1970) was used. Supercomputer simulations may provide a solution here although for practical purposes this calculation is not currently feasible. Solution of high resolution fluid flow problems would thus appear to be best performed with an element-by-element code where matrices are stored in their elementary form. Computation times on an HP9000/735 workstation for the remaining successful simulations are given in Table II.

Table II. Computation times and efficiencies for simulations of a one in one year recurrence interval flood event conducted on an HP9000/735 workstation using a variety of numerical techniques and mesh resolutions

Mesh/solver	No. of nodes	Computation time (min)	Efficiency (time per 1000 nodes per time step)
Low resolution/RMA-2	1200	265	7·361 min
Lowe resolution/SUPG	1200	320	0·593 s
High resolution/SUPG	5600	2188	0·868 s
High resolution/hybrid	5600	2930	1·162 s

Little difference in simulated discharge or inundation extent is shown between TELEMAC-2D calculations made with the SUPG and hybrid numerical solution schemes. The SUPG scheme appears to attain a more convincing initial steady state, but once the rising limb of the hydrograph is reached the hybrid numerical technique produces near-identical discharge predictions. In terms of computation time, the SUPG method is more efficient (see Table II), giving an approximate 28 per cent reduction in computational requirements for this flow problem over the alternative hybrid scheme. Over the entire simulation the SUPG scheme also gave an approximate 20 per cent reduction in the relative error on mass calculated by the model. The Galerkin weighted residual technique employed by RMA-2 correctly predicts the timing but not the nagnitude of the observed discharge peak. Fully implicit Galerkin techniques of the type employed by RMA-2 have been shown by Fourier analysis to have an undesirable damping effect on the solution by reducing the amplitude of simulated waves. It may be that this is a cause of the underprediction of the flood peak by RMA-2 whereby the damping effect would not allow the model to capture the full dynamic range of the hydrograph. Further testing would be needed to determine this conclusively. Despite this, all simulations show a realistic initial decline in discharge magnitude. RMA-2 and TELEMAC-2D similarly estimate maximum inundation extent but differ in both the timing and range predicted. This may partially be a consequence of not controlling for roughness parameterization in the two schemes. In addition, both models commence the simulation with a partially wet floodplain; this implies either that the flow field along the reach prior to the flood event was not a steady state or that the channel topographic resolution was not sufficient to precisely identify bankfull discharge in particular locations. Previous studies with TELEMAC-2D (Hervouet and Janin, 1994) have shown that simulations can be developed with a completely dry mesh and continued until the flood has receded. With RMA-2, however, model instability was noted for floodplain states that were predominantely dry, particularly under conditions of flood wave recession.

Moving from a low to a high resolution finite element mesh, TELEMAC-2D shows relatively little improvement in the accuracy of discharge predictions. The high resolution mesh gives a better fit to the peak discharge; however, timing of the peak and the initial part of the simulation are replicated more closely by the low resolution mesh. This probably reflects a need to refine the calibration for the high resolution mesh instead of merely transferring the previously derived low resolution calibration. This implies that calibrations are, to some extent, discretization specific and non-stationary. Both finite element meshes do, however, produce good results well within the error limits of the data set. In terms of TELEMAC-2D inundation extent, the high resolution mesh predicts a greater range than the low resolution mesh, albeit with similar timing, and is also able

to simulate significantly drier floodplain states. This is largely to be expected given the reduced element size. Uniquely for this flood event, an estimate of maximum flood inundation extent was available (see Figure 5) from an amalgamation of air and ground photo sources and post-event mapping of floodplain trash lines (Simm, 1993). This indicated a maximum flood inundation extent of 68 per cent. This single observation has been included on Figure 4b assuming a measurement error of ±20 per cent and a timing peak inundation of $9 \pm 1{\cdot}5$ h after the start of the event. The timing given for peak inundation is necessarily imprecise given the nature of the data sources used; however, Figure 5 still represents a unique record for this scale of river reach. Ideally, a sequence of synoptic flood inundation extent maps taken hourly through an event would be needed as a starting point for model validation but at present no data set exists. As can be seen from Figure 5, all simulations reported in this study are in or close to the upper portion of the observed inundation extent error band, with simulations on the low resolution mesh showing the best correspondence. This may represent a need to refine the topographic specification used to create the model, although given that only one data point with assumed error bands is available, it is difficult to make any definite statement.

These results indicate that a good solution of the controlling equations is possible even with a rather coarse discretization of the computational domain and that, in terms of computational cost, such calculations can be achieved relatively cheaply. It is likely, however, that lack of an equivalent resolution topographic data set is constraining the performance of the high resolution mesh. In this study, therefore, the high resolution mesh is overspecified relative to topography. If higher resolution topographic data were available it is probable that the high resolution mesh would give improved performance, particularly in relation to spatially distributed predictions of hydraulic variables.

Topographic parameterization accuracy

The second part of this investigation into the numerical solution and parameterization of two-dimensional finite element models for river channel/floodplain flow problems concerns an initial attempt to quantify the impact of topographic parameterization accuracy on simulation results. All acquired data are subject to some degree of error. This is particularly true of topographic data in areas of low lateral bed slope such as floodplain where, unless an intensive surveying programme is implemented, few data exist from published sources and those that are available may be of reduced quality. The prediction of floodplain inundion extent is also a

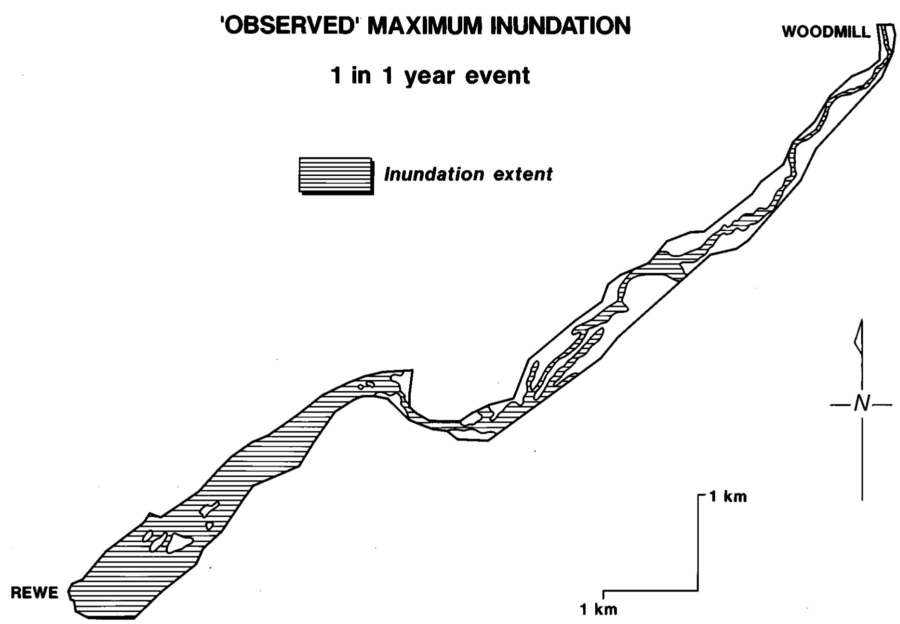


Figure 5. 'Observed' maximum inundation extent for the one in one year recurrence interval flood event shown in Figure 4 (after Simm, 1993)

quantity with a high sensitivity to topographic parameterization, as a small error in the specified bed elevation may have a large impact on the predicted lateral extent of the flow field.

These effects have been examined using a distributed sensitivity analysis to determine spatial and temporal variations in model response to varying topographic parameterization accuracy. Initially, this investigation has been solely conducted using the RMA-2 finite element code in conjunction with the low resolution mesh shown in Figure 2 and the one in one year recurrence interval flood event described above. Four model simulations of this flood event were conducted: a control calculation and three further calculations where all floodplain nodes were lowered by 5, 15 and 20 cm from their original elevation respectively. These values were chosen as they were felt to be representative of the accuracy of topographic parameterization found for floodplain environments. Although a uniform change in floodplain elevation would not be a typical error surface, such an analysis should be capable of giving a preliminary indication of model sensitivity in this respect and provide necessary information for a more sophisticated research design. For each simulation a time sequence of floodplain inundation extent maps was constructed at intervals of 3 h. Then for a series of 20 cross-sections along the reach, approximately 0·5 km apart, the percentage floodplain inundation was determined. The increase in inundation extent at these points caused by a lowering of the floodplain surface could then be calculated and variation of this quantity in both space and time obtained. These data are summarized for the –5, –15 and –20 cm treatments in Figures 6a, b and c.

Figure 6 indicates that certain areas of the mesh are more sensitive to inaccuracies in topographic parameterization than others and, moreover, that this sensitivity is consistent *within* treatments over the course of a dynamic flood simulation. This is evidenced by the strong ridges present in the data, indeed in the initial stages of the simulation some decreases in inundation extent, representing negative sensitivites, are apparent. This is probably a consequence of the dynamic nature of the simulation and illustrates the insights into model performance to be gained from this type of analysis. The passage of a floodwave through the reach can be clearly seen as a trough in the sensitivity surface running from approximately 2·5 h into the simulation at the head of the reach to approximately 10 h at the downreach end. This is an artefact of the analysis method whereby during periods of high floodplain occupancy only small increases in percentage inundation are possible. Correlations between aspects of model geometry such as floodplain width and lateral slope have also been investigated as possible means of explaining some of the pattern in the sensitivity surface, but no significant relationship has been found (Bates and Anderson, 1996). Figure 6 also shows significant variation in space/time response *between* treatments. The results imply that the accuracy of topographic data used to parameterize the model may be more important in some specific locations than others, a conclusion that, potentially, has significant implications for data collection programmes for two-dimensional finite element schemes. It is clear therefore that even such a restricted investigation as outlined here may be able to aid in the design of field data capture programmes and show the potential of the methodology for understanding complex modelling systems.

DISCUSSION

This paper has successfully applied two two-dimensional finite element codes to a relatively long (11 km) river reach with the specific aim of investigating the numerical solution, discretization and parameterization of this class of scheme for river flood simulation problems. In terms of the relative merits of the two schemes, the RMA-2 code incurs the lower computational cost due to the use of a relatively long time step. This may, however, give a high Courant number. Despite the fact that model stability is not affected by such constraints for an implicit scheme, the physical basis of the Courant number means that for a good quality solution it is often desirable to keep within a reasonable limit. For this reason the TELEMAC-2D code may provide a more accurate solution of the controlling equations and this would appear to be borne out by the data presented in Figure 4a.

Two separate issues arising from this study should be highlighted. Firstly, this research has indicated a potentially significant interaction between the resolution of the topographic data used to establish the model and the resolution of the finite element mesh employed. Both mesh construction and topographic data capture have significant resource requirements which the modelling process should seek to minimize. A primary requirement for future research is therefore the development of an ability to identify mesh/topography

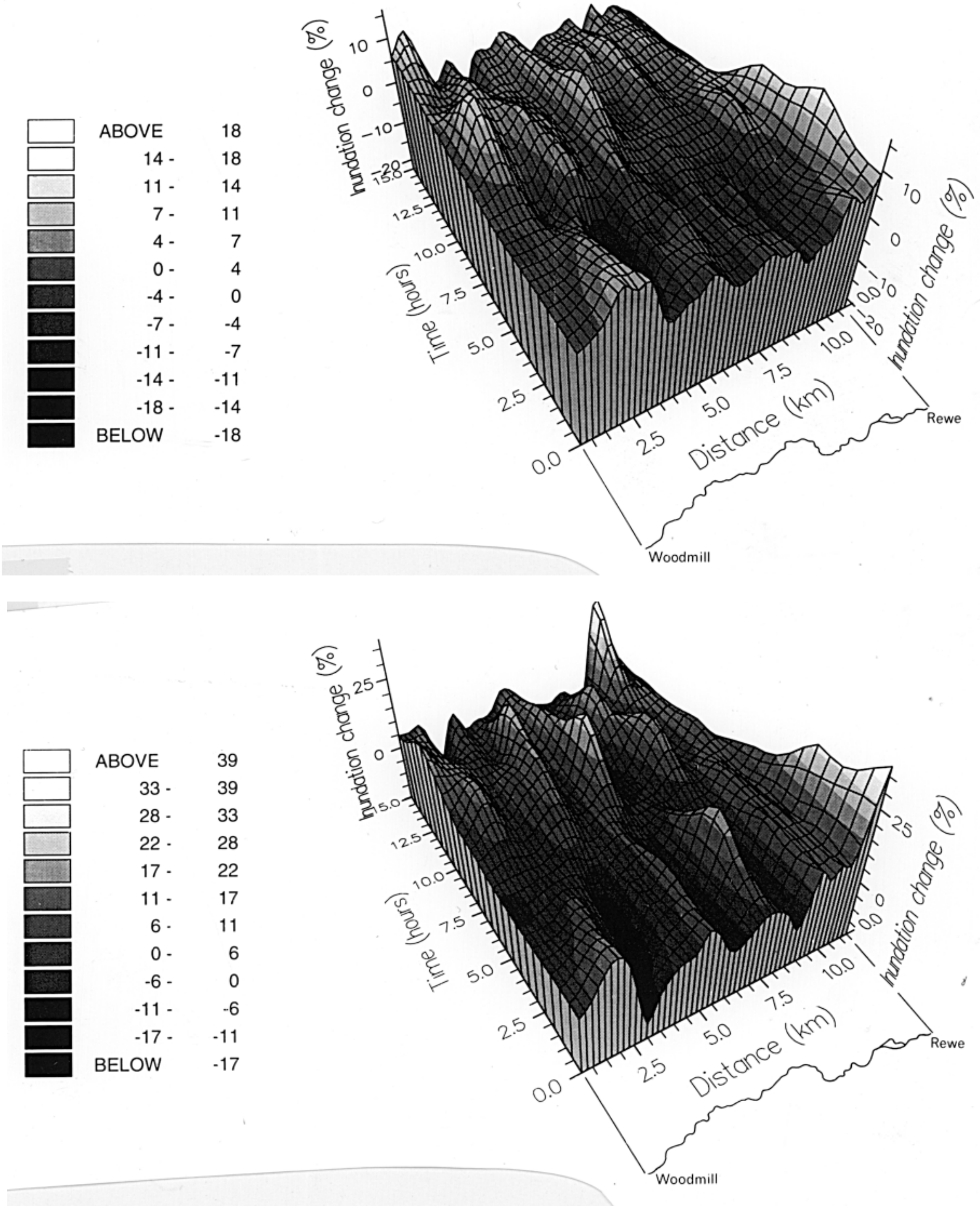
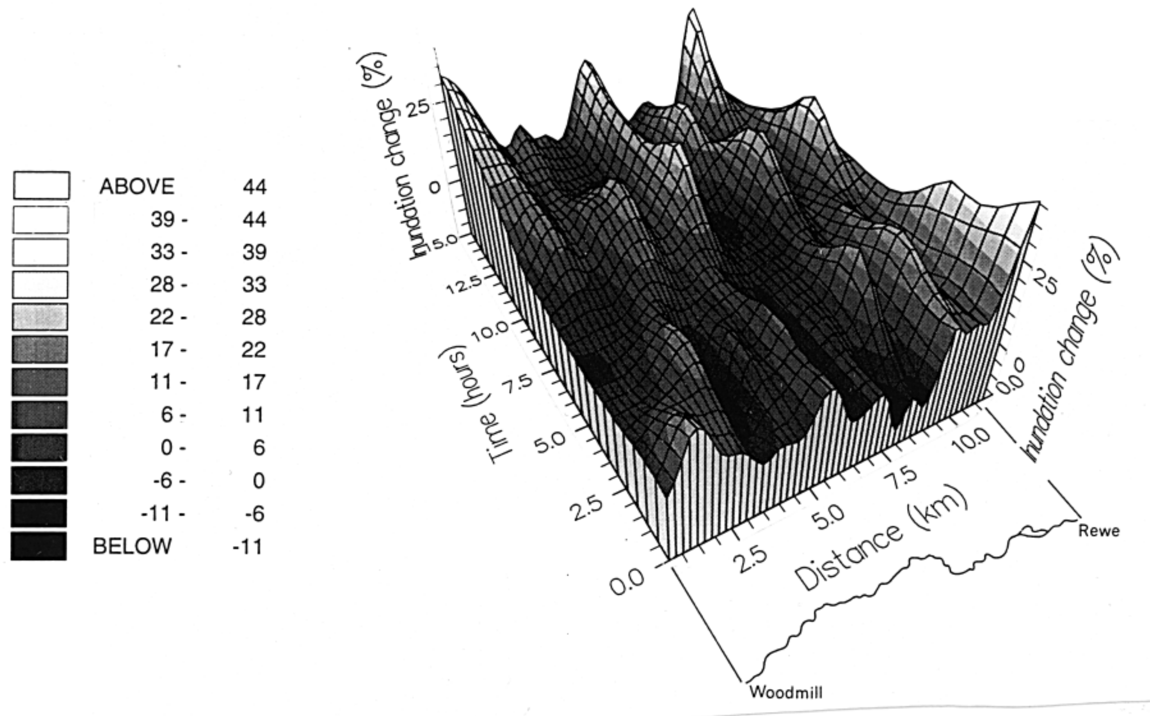


Figure 6. Space/time distributed sensitivity surfaces of model response to variations in topographic parameterization accuracy. Shown in three three-dimensional surfaces of percentage change in simulated inundation extent for global decreases in floodplain nodal elevations of 5 (a), 10 (b) and 20 cm (c) respectively, in terms of distance along reach from Woodmill (upstream) to Rewe (downstream) on the *x* axis and time in hours from the start of the one in one year recurrence interval event shown in Figure 4a



combinations which do not overspecify either component yet meet application objectives. These have yet to be defined for floodplain geomorphology or water quality studies. Secondly, a high resolution space/time sensitivity analysis has shown a complex and significant response to the topographic data accuracy used in model construction. Commonly in such models, boundary friction is regarded as the most important calibration parameter. This research raises the possibility that other aspects of the model parameterization may also be important at particular spatial and temporal locations or for particular model predicted variables. An understanding of these issues appears essential to achieve a more explicit comprehension of model parameterization and calibration impacts on model validation. The current investigation therefore suggests two areas for future research: further space/time distributed sensitivity analysis to establish the links between parameter sensitivity and model response; and secondly, on the basis of this information, to begin a more comprehensive validation programme for this class of model.

The necessity to acquire a wider set of information concerning model behaviour and performance of the type presented in this paper has been established. At present such information is relatively unsophisticated and typically only allows model construction and calibration to be optimized for single variables at specific locations on the model distributed network, for example reach downstream outlet discharge. Model response to possible optimization parameters has, however, been shown to be more complex than this restricted view implies. If high resolution numerical models of the type presented in this paper are to achieve their potential ability to make spatially and temporally distributed predictions of many variables simultaneously, information detailing this complexity is a key requirement. Furthermore, complex model response to parameter variation implies that caution should be shown when considering validation studies based on a limited number of single variable optimizations.

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